The Power of Quadruple Precision Accuracy

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September 10, 2009

1 Introduction

In fields such as numerical relativity, the accuracy of obtained solutions and values is often more important than the time it takes to get these values. Once the desired level of precision has been reached, then acceleration techniques can begin to be implemented. Modern hardware accelerators have significant difficulty even dealing with numbers of the double precision type. Since these data types reside in two memory locations (allowing for the added accuracy as opposed to single precision), the computations become extremely time consuming. Due to the significant amount of computing time, often times accuracy is sacrificed in order to provide a somewhat reasonable compiling time. If a method of using a data type as accurate as quadruple precision were to be developed, the benefit of this development would be felt throughout the entire computational science-related world.

2 A New Level of Accuracy

2.1 Single Precision

Single Precision data types are known to take the least amount of time with respect to computational times. These types of variables are commonly known as floats in the "C" programming language. Single precision elements occupy 32 bits (4 bytes). The significand has a precision of 24 bits, which coverts to roughly 7 decimal places of accuracy. For standard applications, floats are more than accurate enough. But when dealing with physics and complex math, many more decimal places of accuracy are needed.

2.2 Double Precision

Double precision data types provide many more places of accuracy in comparison to single precision types. As previously mentioned, these types of variables reside in two separate memory locations, which is why computations using these types take much longer than those of the single precision variety. Double precision variables occupy 64 bits (8 bytes) and its significand has an accuracy of 53 bits, or 16 decimal digits.

2.3 Quadruple Precision

Quadruple precision data types are the most accurate types in use today. Other, more accurate types have been implemented, but as the level of precision increases, the amount of computing power and time required to utilize these variables increases exponentially at any level of precision higher than quadruple. As it currently stands, very few processors can harness the power of quadruple precision accuracy. These data types support 128 bits of accuracy which is equivalent to 32-33 places of accuracy depending on the application. With this kind of accuracy, even astronomers and physicists cannot complain. More accuracy is always desirable, but for the amount of time it takes to compute, quadruple precision variables provide the most bang for the buck.

2.4 Magnitude of accuracy

General libraries have been developed for use with CPU's in a variety of different languages. One of the more popular ones, the High Precision Arithmetic library allows numbers in the range of $2^{-16383} < x < 2^{16384}$ or $1.19 * 10^{4932} > x > 1.68 * 10^{-4932}$ (www.nongnu.org). If you care to do the math there, the range of representable numbers is almost mind blowing.

3 Relevance to Numerical Relativity

Numerical Relativity is a very delicate field. When dealing with objects or particles that are extremely massive or nearly massless, accuracy is a necessity to obtain an accurate description of the particle or object. A crude example of the necessity of this accuracy can be shown with this gedanken (thought experiment). Consider a comet, that is the size of Texas. If such a comet were to strike Earth, the results would be catastrophic. Some physicists in a computer laboratory discover a comet of these proportions in a far off galaxy, and notice that it is heading ”towards” our solar system. The scientists immediately begin crunching numbers to determine if the comet is an actual threat to Earth. It is here in these calculations, where accuracy becomes a priority. With the comet being millions of miles away, significant accuracy is needed to approximate the comets trajectory. If the scientists were to use a single precision data type, their calculation would be accurate to 7 decimal places. Double precision data types would yield correct accuracy to 16 digits, and quadruple precision would be accurate to roughly 32 digits. In a scenario such as this, getting a fast answer would provide a completely irrelevant result. A difference of .00000000001 in the angle at which the comet is traveling, over millions of miles, would provide a final location very far off from where the comet actually ends up. For every day use, speed is much greater than precision. But when it comes to the fate of the planet (in this case) precision has to take precedence over speed.

3.1 Modern Techniques

Today, mathematically intensive equations and systems are calculated via parallel computing. This style of computing attempts to get many things done at once. A good analogy to explain the benefits of this type of computing can be done with a homework assignment. If a teacher assigns a class 50 problems of homework, and each problem takes 1 hour to do, the end result is usually a very unhappy student. One student alone would have to do each problem, one at a time. But in a class, there are several students, all with the same homework assignment. If each student does one problem, the total amount of time spent on the assignment as a whole would still be 50 hours, but individually, the amount of work would be significantly reduced. Upon completion, these students can converge and compare results. This convergence at the end of the task allows the students to get the same end result, but in a fraction of the time. This is exactly how parallel computing works. If a problem can be broken up into many pieces and ran in parallel, the computation time can often be decreased by over an order of magnitude. With this type of speedup possible, time intensive applications (such as the Teukolsky equation) now become easier than ever to solve. And, as we all know, more problems solved= more knowledge on the topic.